

# CATEGORY THEORY

## CATEGORY III - GRAPHS

PAUL L. BAILEY

### 1. GRAPHS

**Definition 1** (Objects). Let *graph*  $(V, \mathcal{E})$  consists of a set  $V$  together with a collection of subsets  $\mathcal{E} \subset \mathcal{P}(V)$  such that each member of  $\mathcal{E}$  contains exactly two elements. The members of  $V$  are called *vertices* and the members of  $\mathcal{E}$  are called *edges*.

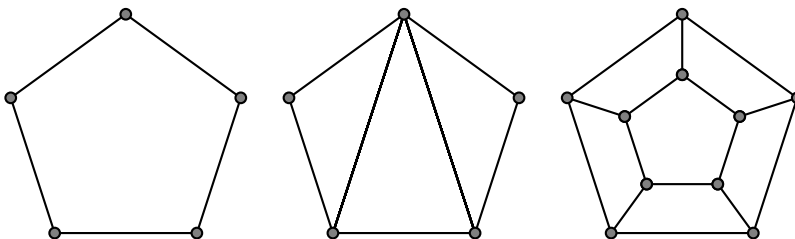
**Definition 2** (Subobjects). Let  $(V, \mathcal{E})$  be a graph. A *subgraph* of  $V$  consists of a set  $W \subset V$  and a set  $\mathcal{F} \subset \mathcal{E}$  such that  $\{w_1, w_2\} \in \mathcal{F}$  implies  $w_1, w_2 \in W$ .

**Definition 3** (Morphisms). Let  $(G, \mathcal{E})$  and  $(H, \mathcal{F})$  be graphs. A function  $f : G \rightarrow H$  is called *edge preserving* if

$$\{v_1, v_2\} \in \mathcal{E} \Rightarrow \{f(v_1), f(v_2)\} \in \mathcal{F}.$$

The identity map on  $V$  is edge preserving, and the composition of edge preserving functions is edge preserving. Thus, graphs with edge preserving maps form a category.

**Problem 1.** Describe the automorphism groups of each of these graphs.



DEPARTMENT OF MATHEMATICS, BASIS SCOTTSDALE  
 Email address: pbailey@basised.com